A New Keynesian Model with Robots: Implications for Business Cycles and Monetary Policy*

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Abstract

This paper examines the effects of labor-replacing capital, which is referred to as robots, on business cycle dynamics using a New Keynesian model with a role for both traditional and robot capital. This study finds that shocks to the price of robots have effects on wages, output, and employment that are distinct from shocks to the price of traditional capital. Further, the inclusion of robots alters the response of employment and labor's share to total factor productivity and monetary policy shocks. The presence of robots also weakens the correlation between human labor and output and the correlation between human labor and labor's share. The paper finds that monetary policymakers would need to place a greater emphasis on output stabilization if their objective is to minimize a weighted average of output and inflation volatility. Moreover, if policymakers have an employment stabilization objective apart from their output stabilization objective, they would have to further focus on output stabilization due to the deterioration of the output-employment correlation.

Keywords: robotization, labor's share of income, monetary policy, business cycle fluctuations
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Introduction

Recent developments in artificial intelligence, computer vision, and other technologies raise the prospect that machines will replace human labor in many jobs until recently thought to be immune to the forces of mechanization. A number of recent commentators (Brynjolfsson and McAfee (2014), Krugman (2012), and Summers (2013), to name a few) have warned that the latest round of automation has had an adverse impact on wages and employment, and that the situation will only get worse as robots and other new technologies become more plentiful and more powerful substitutes for human labor.

While the long-run effects of robotization on wages and employment have received a great deal of attention, the effect of robotization at business cycle frequencies has not been studied. These effects, however, are potentially important. Casual inspection of Figure 1 shows that labor's share fluctuates over the business cycle. Up to the 1990s, the typical pattern was for labor's share to fall during the recession and in the early stages of recovery, then rise during the recovery and expansion period. In the last two business cycles, however, labor's share has fallen particularly strongly during the recession and early recovery period and failed to climb during the expansion. This pattern is consistent with robotization playing a greater role during recessions and expansions than in earlier business cycles.

[Figure 1 about here.]

This paper embeds robot capital into a medium-sized New Keynesian model along the lines of Smets and Wouters (2005) in order to examine the effects of robotization at business cycle frequencies. In the model output is produced using human labor, traditional capital, and robot capital using a nested constant elasticity of substitution (CES) production function. Robot capital is distinguished from traditional capital by its elasticity of substitution with human labor: traditional capital is a complement to human labor whereas robot capital substitutes for human labor. Consequently, all else equal, investment in robot capital reduces the relative demand for human labor with adverse effects on employment and wages.

At the same time, robotization may have effects in general equilibrium effects that offset this reduction in human labor. There are at least three potential mitigating factors. First, the introduction of new machinery increases the marginal product of complementary types of physical capital, induces investment in traditional capital which is complementary to human labor. Second, demand for robots increases demand for human labor in their production. Finally, the employment effects of the introduction of labor-displacing machinery will depend on the extent to which greater productivity is matched by an increase in aggregate demand. This paper incorporates all of these offsetting effects.

This paper explores three types of issues. First, it asks how investment specific shocks that lower the price of robot capital differ in their economic effects from shocks to total factor productivity (TFP) or to the price of traditional capital investment. The key distinction is that whereas TFP shocks or a decrease in the cost of traditional capital resulting in an increase in investment has the direct effect of increasing the marginal product of labor and hence increasing labor demand, a decrease in the cost of robot capital that causes increased investment in robot capital may subsequently crowd out human labor. This paper examines the implications for wages, employment, and labor's share of income.

Second, this paper documents the implications of robotization for the cyclical behavior of labor's share. A number of papers have found that output and labor's share are negatively correlated, with estimates ranging from -0.11 (Choi and Ríos-Rull, 2009) to -0.71 (Ambler and Cardia, 1998). This paper shows that robotization can account for this negative correlation.

Lastly, this paper explores the implications of the presence of robot capital for the conduct of monetary policy. The presence of robots alters the relationship between output and employment over the business cycle. The effect of robot investment on wages has an effect on current and expected future marginal cost, thereby altering the cyclical behavior of inflation as well. While a formal analysis of optimal monetary policy in this model is left for future research, this paper identifies the general effects of robots on the emphasis a monetary policy authority would place on output and inflation stabilization objectives in its monetary policy rule if its objective is to minimize a weighted average of output and inflation volatilities.

This paper explores these issues in a model calibrated to match the current importance of robot capital in the United States economy. Given the current trends in technology, however, it also asks the questions posed above in a model in which robot capital plays a considerably larger role than it does today.

Model

The model builds upon Smets and Wouters (2005) by incorporating robots via a nested CES production function. The model consists of a representative final good firm, a continuum of intermediate good firms, a continuum of labor unions, a representative household, and a monetary authority. The final good firm produces output using intermediate goods. Intermediate goods firms produce intermediate goods using capital, robots, and an aggregate of differentiated labor supplied by labor unions. Intermediate goods prices and wages are set by monopolistically competitive firms and labor unions respectively under Calvo (1983) pricing. The household owns the firms, purchases consumption goods, and places its savings in the form of bonds, traditional capital, and robots. The household also supplies undifferentiated human labor to the labor unions which transform it into differentiated labor which is hired by the intermediate good firms.

Production

Final Good Firms

Competitive final good producers purchase differentiated intermediate goods from intermediate goods producers to produce a composite final good. The production of a final good y_t

uses intermediate goods $y_t(i)$, $i \in [0,1]$, using the production function:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\varepsilon_y}} \, di \right)^{\varepsilon_y},$$

where $\varepsilon_y > 1$. Profit maximization and perfect competition imply the demand for intermediate good i is

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{\varepsilon_y}{1-\varepsilon_y}} y_t,$$

where the final good price index is

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\varepsilon_y}} \, di \right)^{1-\varepsilon_y}.$$

Intermediate Good Firms

There is a continuum of intermediate good firms owned by the representative household. They are indexed by $i \in [0,1]$. Each period a fraction $(1-\lambda_y) \in [0,1]$ of the intermediate good firms can re-optimize its price. The remainder sets price according to the indexation rule:

$$P_t(i) = {\pi_{t-1}}^{\eta_y} P_{t-1}(i),$$

where $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ is the gross inflation rate in the economy, and η_y is the inflation indexation parameter. The intermediate good producer i has access to a constant elasticity of substitution production technology:

$$y_t(i) = z_t \left[\theta_k \tilde{k}_t(i)^{\alpha} + (1-\theta_k) \ell_t(i)^{\alpha} \right]^{\frac{1}{\alpha}},$$

where z_t is total factor productivity, $\tilde{k}_t$ is utilization-adjusted traditional capital, and ℓ_t is composite labor input. ℓ_t is an aggregation of utilization-adjusted robots $\tilde{a}_t$ and human labor input n_t:

$$\ell_t(i) = \left[\theta_a \tilde{a}_t(i)^{\phi} + (1-\theta_a) n_t(i)^{\phi}\right]^{\frac{1}{\phi}}.$$

The use of the CES production function rather than the traditional Cobb-Douglas function enables this paper to distinguish between traditional capital, which complements human labor at the level of the industry, and robot capital, which substitutes for human labor. The curvature parameters α and ϕ determine the degree of substitutability or complementarity between the two forms of capital and human labor. The elasticity of substitution between traditional capital and composite labor is $\frac{1}{1-\alpha}$ while the elasticity of substitution between composite and human labor is $\frac{1}{1-\phi}$. This paper will maintain the assumption that $\alpha < 0$, implying that the elasticity of substitution between traditional capital and composite labor is less than one, hence traditional capital and composite labor are gross complements. It will assume that $\phi > 0$, implying that the elasticity of substitution between robot capital and human labor is greater than one, hence robot capital and human labor are gross substitutes.

The intermediate good firms rent capital and robots from the households and hire labor input from labor unions. Total factor productivity z_t follows the stochastic processes:

$$\ln z_t = \rho_z \ln z_{t-1} + \varsigma^z \eta_t^z, \;\; \eta_t^z \sim \text{i.i.d. } \mathcal{N}(0,1).$$

Intermediate good firm i's cost minimization problem is:

$$\min_{\{\tilde{k}_t(i), \tilde{a}_t(i), n_t(i)\}} P_t r_t^k \tilde{k}_t(i) + P_t r_t^a \tilde{a}_t(i) + W_t n_t(i),$$

subject to production technology:

$$y_t(i) = z_t \left\{\theta_k \tilde{k}_t(i)^{\alpha} + (1-\theta_k)\left[\theta_a \tilde{a}_t(i)^{\phi} + (1-\theta_a) n_t(i)^{\phi}\right]^{\frac{\alpha}{\phi}}\right\}^{\frac{1}{\alpha}},$$

where r_t^k, r_t^a, and W_t are the capital rental rate, robot rental rate, and nominal wage which firms take as given. It can be shown that, taking factor prices and productivity as given, each intermediate good firm's cost-minimization problem implies the marginal cost of production is constant with respect to the level of output and is independent of i. Since marginal cost is identical across firms, the i subscript is dropped in the price adjustment problem below.

Each price re-optimizing firm chooses price $\tilde{P}_t$ to maximize its expected profit. Since the intermediate good firms are owned by the household, they discount future profits using the household's stochastic discount factor. The price-adjusting intermediate good firm's price-setting problem can be expressed as:

$$\max_{\tilde{P}_t} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\lambda_y)^s \frac{\Lambda_{t+s}}{P_{t+s}} \left(\tilde{P}_t \prod_{k=1}^{s} {\pi_{t-1+k}}^{\eta_y} - MC_{t+s} \right) \tilde{y}_{t+s},$$

where Λ_{t+s} is the household's marginal utility; MC_{t+s} is the marginal cost of production; and $\tilde{y}_{t+s}$ is the period $t+s$ demand for an optimizing firm's output, assuming its last price adjustment was in period t.

Household

The representative household owns the stock of physical capital k_t and robots a_t and makes investment decisions i_t^k and i_t^a. The household chooses the levels of utilization for capital and robots μ_t^k and μ_t^a which incur utilization costs $\Psi_k(\mu_t^k)$ and $\Psi_a(\mu_t^a)$. The household purchases nominal bonds D_t and receives dividend payments Div_{t+s} from its ownership of intermediate good producers.

The household's lifetime utility is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(c_{t+s} - hc_{t-1+s})^{1-\gamma}}{1-\gamma} - \kappa_n \frac{{n_{t+s}}^{1+\sigma}}{1+\sigma} \right].$$

Its budget constraint is:

$$
\begin{aligned}
& c_{t+s} + \frac{D_{t+s}}{P_{t+s}} + i^k_{t+s} + i^a_{t+s} + \Psi^k\left(\mu^k_{t+s}\right)k_{t+s} + \Psi^a\left(\mu^a_{t+s}\right)a_{t+s} \leq \\
& \frac{W_{t+s}}{P_{t+s}}n_{t+s} + r^k_{t+s}\mu^k_{t+s}k_{t+s} + r^k_{t+s}\mu^a_{t+s}a_{t+s} + R_{t-1+s}\frac{D_{t-1+s}}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}}.
\end{aligned}
$$

Here β is the discount factor; c_{t+s} is the level of consumption; h captures habit formation; γ is the coefficient of relative risk aversion; κ_n and σ capture the disutility of work.

In addition to the variables already described above, r^k_{t+s} and r^a_{t+s} are the rental rates for each unit of effective capital $\mu^k_{t+s}k_{t+s}$ and effective robot $\mu^a_{t+s}a_{t+s}$, respectively. The households earn gross interest R_{t+s} for each unit of nominal bonds they hold.

The capital and robot laws of motion are:

$$
\begin{aligned}
k_{t+1+s} &= (1-\delta_k)k_{t+s} + \varepsilon^k_{t+s}\left[1 - \mathcal{S}_k\left(\frac{i^k_{t+s}}{i^k_{t-1+s}}\right)\right]i^k_{t+s}; \\
a_{t+1+s} &= (1-\delta_a)a_{t+s} + \varepsilon^a_{t+s}\left[1 - \mathcal{S}_a\left(\frac{i^a_{t+s}}{i^a_{t-1+s}}\right)\right]i^a_{t+s},
\end{aligned}
$$

where $\mathcal{S}_k(\cdot)$ and $\mathcal{S}_a(\cdot)$ are capital and robot adjustment cost functions, and ε^k_{t+s} and ε^a_{t+s} are investment-specific shocks. The shocks follow the autoregressive processes:

$$
\begin{aligned}
\ln\varepsilon^k_t &= (1-\rho_k)\ln\bar{\varepsilon}^k + \rho_k\ln\varepsilon^k_{t-1} + \varsigma^k\eta^k_t,\ \ \eta^k_t \sim \text{i.i.d. } \mathcal{N}(0,1); \\
\ln\varepsilon^a_t &= (1-\rho_a)\ln\bar{\varepsilon}^a + \rho_a\ln\varepsilon^a_{t-1} + \varsigma^a\eta^a_t,\ \ \eta^a_t \sim \text{i.i.d. } \mathcal{N}(0,1),
\end{aligned}
$$

where $\bar{\varepsilon}^k$ and $\bar{\varepsilon}^a$ are steady state values of the inverse of the price of traditional and robot capital investment goods which are normalized to 1 in the benchmark model.

The specifications for the utilization cost and investment adjustment cost functions are standard and are specified in the Online Supplemental Appendix.

Labor Unions

There is a unit measure of labor unions indexed by $j \in [0,1]$. The representative household supplies undifferentiated labor to labor unions, which in turn supply type j labor $n_t(j)$. Differentiated labor is then combined using:

$$n_{t+s} = \left(\int_0^1 n_{t+s}(j)^{\frac{1}{\varepsilon_n}} \, dj \right)^{\varepsilon_n},$$

to form human labor input. Intermediate good producers hire n_t. A competitive market for n_t yields the demand for type j labor:

$$n_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{\frac{\varepsilon_n}{1-\varepsilon_n}} n_t;$$

and the nominal wage for n_t is:

$$W_{t+s} = \left(\int_0^1 W_{t+s}(j)^{\frac{1}{1-\varepsilon_n}} \, dj \right)^{1-\varepsilon_n}.$$

Each period a fraction $1 - \lambda_n$ of the unions are allowed to renegotiate their wages. Representing the household, unions maximize the utility of the household subject to the labor demand and the budget constraints of the household.

An optimizing union j's problem is:

$$\max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\lambda_n)^s \, \varepsilon^b_{t+s} \left\{ \left[\frac{(c_{t+s} - hc_{t-1+s})^{1-\gamma}}{1-\gamma} - \kappa_n \frac{\tilde{n}_{t+s}(j)^{1+\sigma}}{1+\sigma} \right] \right\},$$

subject to labor demand and the household's budget constraint.

Monetary Authority

The monetary authority sets the short-term nominal rate according to the following rule:

$$\left(\frac{R_t}{R^*}\right) = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\rho_\pi} \left(\frac{y_t}{y^*}\right)^{\rho_Y}\right]^{1-\rho_R} \varepsilon_t^{mp},$$

where R^* is the steady-state nominal interest rate, π^* is the steady-state inflation rate, y^* is the steady-state level of output, and ε_t^{mp} is the monetary policy shock. ε_t^{mp} follows the stochastic process:

$$\ln \varepsilon_t^{mp} = \rho^{mp} \ln \varepsilon_{t-1}^{mp} + \varsigma^{mp}\eta_t^{mp},\ \eta_t^{mp} \sim \text{i.i.d. } \mathcal{N}(0, 1).$$

Resource Constraint

The resource constraint

$$y_t = c_t + i_t^k + i_t^a + \Psi(\mu_t^k)k_t + \Psi(\mu_t^a)a_t$$

closes the model.

The equilibrium equations representing the solution to the model are shown in the Online Supplemental Appendix.

Results

Calibration

The model is calibrated as shown in Table 1. Details of the calibration procedure are reported in the Online Supplemental Appendix. Each period in the model is one quarter. This paper follows convention and set the time preference rate β to 0.99 and the depreciation rate for traditional capital δ_k to 0.02 (see, e.g., Kydland, 1995, p. 148). It sets the depreciation rate on robot capital δ_a to 0.04. The higher depreciation rate reflects the more rapid decline in the value of software and advanced-technology equipment relative to machinery and structures. Krusell et al. (2000) make a similar assumption in a model that distinguishes between structures and equipment.

The utility function parameters γ, σ, κ_n, and h; capital investment adjustment and utilization parameters κ_k, and ν_k; price and wage adjustment parameters λ_y, η_y, λ_n, and η_n are set to the median parameter estimates for the United States from the first quarter of 1983 to the second quarter of 2002 in Smets and Wouters (2005). In the absence of any information to suggest otherwise, this paper sets the robot capital investment adjustment and utilization parameters κ_a and ν_a to the same values as those for traditional capital. The markup parameters for intermediate goods and labor ε_y and ε_n are set equal 1.23 and 1.15 respectively as in Justiniano et al. (2010).[1]

Next this paper calibrates the weights on traditional capital and robot capital in the production function and composite labor function θ_k and θ_a as well as the corresponding curvature parameters α and ϕ. First, this paper specifies a "no robot" scenario, one in which traditional capital and robots share a common curvature parameter that matches the elasticity of substitution between capital and labor in the data.[2] According to Chirinko (2008) the consensus in the empirical literature is that the elasticity of substitution lies between 0.4 and 0.6. Taking the midpoint of this range implies curvature parameters of $\alpha = \phi = -1$. Values for $\theta_k = 0.5459$ and $\theta_a = 0.1251$ are chosen to match the capital-output ratio and robot-to-capital ratio in the data.

Throughout the analysis this paper compares the no robot scenario described above to two alternative scenarios. The first is the "weak robot" scenario which sets $\phi = 0.25$ while holding all other parameters equal to their no robot scenario values. This implies that robots

[1]The authors acknowledge this model is not directly comparable to those in Smets and Wouters (2005) and Justiniano et al. (2010); estimation of this model might produce different parameter values. This paper leaves estimation of the model as the subject of future research.

[2]This implies that the difference between capital and "robots" in this specification is the difference in their depreciation rates.

and human labor are substitutes, with an elasticity of substitution of $\frac{4}{3}$. The second is the "strong robot" scenario which sets $\phi = 0.5$, implying a higher elasticity of substitution of 2.

Finally this paper needs to specify the stochastic processes for the TFP, capital investment, robot investment, and monetary policy shocks. It leaves a full-scale estimation for future work and opt for a cruder approach of estimating the stochastic processes separately using macroeconomic data. (The details of the estimation procedure can be found in the Online Supplemental Appendix.) Table 1 of this paper lists these estimated stochastic processes, along with the calibrated parameters specified above. The model is solved using a first order perturbation method.

[Table 1 about here.]

Business Cycle Effects

The effect of shocks on real wages, employment, labor's share, and other key variables at business cycle frequencies is examined in this section. As noted above, this paper compares three specifications: no robots ($\phi = -1$); weak robots ($\phi = 0.25$); and strong robots ($\phi = 0.5$).

The wage rate is given by the firm's first order condition for human labor which is expressed below. The real wage is an increasing function of TFP, the output to composite labor ratio, and the composite labor to human labor ratio:

$$w_t = (1-\theta_k)(1-\theta_a) mc_t {z_t}^{\alpha} \left(\frac{y_t}{\ell_t}\right)^{1-\alpha} \left(\frac{\ell_t}{n_t}\right)^{1-\phi}. \tag{1}$$

Labor's share, $LS_t \equiv \frac{w_t n_t}{y_t}$, is:

$$LS_t = (1-\theta_k)(1-\theta_a) mc_t {z_t}^{\alpha} \left(\frac{y_t}{\ell_t}\right)^{-\alpha} \left(\frac{\ell_t}{n_t}\right)^{-\phi}. \tag{2}$$

Labor's share is an increasing function of the output to composite labor ratio when $\alpha < 0$

and a decreasing function of the composite labor to human labor ratio when $\phi > 0$.[3] Note that the output to composite labor ratio $\frac{y_t}{\ell_t}$ increases with the capital to composite labor ratio $\frac{k_t}{\ell_t}$, and that the composite labor to human labor ratio $\frac{\ell_t}{n_t}$ is an increasing function of the robot to human labor ratio $\frac{a_t}{n_t}$. In the subsequent analysis this paper will focus on the role these two ratios—capital to composite labor $\frac{k_t}{\ell_t}$ and robot to human labor $\frac{a_t}{n_t}$—have on the real wage and labor's share of income.

In equations (1) and (2), an increase in marginal cost or TFP increases real wages and labor's share. An increase in the capital to composite labor ratio and/or robot to human labor ratio increases the real wage. When $\alpha < 0$ and $\phi > 0$, as in the two specifications of interest in this paper, an increase in the capital to composite labor ratio increases labor's share while an increase in the robot to human labor ratio reduces labor's share. Intuitively, increases in either the capital to composite labor ratio or the robot to human labor ratio raises the marginal product of human labor which in turn increases the real wage. Further, given that capital and human labor are complements, human labor rises with an increase in the capital to composite labor ratio, leading to an increase in labor income. On the other hand, while the real wage rises when the robot to human labor ratio increases, human labor falls when the robot to human labor ratio rises leading to a smaller labor share of income.

Figures 2 to 5 show the response of key variables to one percentage point increases in the four disturbance terms—traditional capital, robot capital, TFP, and the nominal interest rate. The distinctive effects of robot capital are most apparent when one compares a shock to the price of traditional capital shown in Figure 2 with a shock to the price of robot capital shown in Figure 3. In each of these cases movements in marginal cost are minimal and TFP is constant, so movements in wages and labor's share are dominated by the capital to composite labor ratio and the robot to human labor ratio.

A reduction in the price of traditional capital causes the household to increase investment in traditional capital and substitute away from robot capital. The increased investment

[3] In other words, when traditional capital and composite labor are complements and when robots and human labor are substitutes.

demand increases employment, output, and the real wage. The robot to human labor ratio falls and the capital to composite labor ratio rises, leading to an increase in labor's share. The increase in labor's share is greater the more substitutable are robots and human labor. By contrast, while a decrease in the price of robot capital also increases employment, wages, and output, it increases the robot to human labor ratio and reduces the capital to composite labor ratio, causing labor's share to fall. Again, the decline in labor's share is considerably larger in the high substitutability case.

Figure 4 shows the response to a TFP shock. As is common in models with nominal rigidities, a positive TFP shock reduces demand for all three factors of production in the short run because aggregate demand does not rise as much as potential output. While the reduction in input demand would lower rental rates and wages, wage rigidity along with a lower aggregate price level raises the real wage while the rental rates fall. This makes human labor relatively more expensive which induces firms to substitute away from human labor in favor of both types of capital. This raises both the capital to composite labor ratio and the robot to human labor ratio. Overall, the rise in robot to human labor ratio and drop in the marginal cost of production outweigh the rise in capital to composite labor ratio, leading to a fall in labor's share of income. Note that the greater the elasticity of substitution between robots and human capital, the larger is the reallocation from human labor to robot capital and the greater is the decline in labor's share.

Finally, Figure 5 shows the response to a nominal interest rate shock. A reduction in the nominal rate increases demand for output and therefore also for traditional capital, robots, and human labor. Factor prices rise in response to increased demand. In the period immediately following the shock, rental rates rise by more than real wages due to wage rigidity, leading firms to increase employment of human labor relative to traditional capital and robots. Therefore both the capital to composite labor ratio and robot to human labor ratio fall. Over time, however, accumulation of both types of capital reverses these effects and the ratios rise. The net effect is for labor's share to rise in the period immediately following the decrease in interest rates. The increase in labor's share is largest in the strong robot case. In this case the reversal in labor's share is also more dramatic, with the net effect on labor's share becoming negative after about 14 quarters.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

Volatility and Correlations

The classic real business cycle model with a Cobb-Douglas production function yields a labor's share that is constant over the business cycle. As noted above, however, the contemporaneous correlation between output and labor's share is negative. A number of authors have explored deviations from the textbook real business cycle model that can replicate this feature of the data, including Ambler and Cardia (1998) (imperfect competition in product markets); Gomme and Greenwood (1995) (optimal contracting between workers and entrepreneurs); Hansen and Prescott (2005) and Choi and Ríos-Rull (2009) (non-Walrasian labor markets). The purpose here is not to attempt the match the observed correlation in the data, but to examine the effect of robots on these correlations.

This paper computes the analytical correlations between output, hours, and labor's share in each of the three specifications of the model from the transition equations implied by the solution to the model. The results are summarized in Table 2. The first column shows the correlation between labor's share and output and output and employment when the model is driven only by TFP shocks. Within each cell the correlation for the no robots, weak robots, and strong robots specifications is shown. The second through fourth columns show the same correlations when the model is driven only by shocks to the capital investment, robot investment, and the nominal interest rate respectively. In the last column all of the shocks are operative, with the relative variances of the shocks given by the Table 1. The data correlation coefficients come from the quarterly United States real gross domestic product, nonfarm business labor's share, and the nonfarm business hours, de-trended using an Hodrick-Prescott filter with a standard smoothing parameter of 1,600 U.S. Bureau of Economic Analysis (2019); U.S. Bureau of Labor Statistics (2019a,b).[4]

[Table 2 about here.]

The main result is that in the model with all shocks, increasing the elasticity of substitution between robot capital and human labor turns what would be a weak positive correlation between labor's share and output (0.169) into a weak negative correlation (−0.220). This is largely due to a weakening of the correlation between output and employment (from 0.844 to 0.613).

The largest changes in correlations as robots are introduced come through the effect of robot price shocks. Fluctuations in the price of traditional capital and fluctuations in the price of robot capital in the no robots case induce a strong positive correlation between output and hours (0.929 and 0.902 respectively). In the strong robot case, however, the correlation of output and hours becomes negative (−0.143). This change in sign helps explain why the correlation between labor's share and output induced by shocks to the price of robot capital falls from 0.399 in the no robot case to −0.787 in the strong robot case. The introduction of robots has more modest effects on the correlation of labor's share and output arising from the other shocks.

This exercise suggests that the prevalence of robots weakens the relationship between labor's share and output at business cycle frequencies, primarily by weakening the relationship between output and employment. This phenomenon is more pronounced if business cycle fluctuations are driven by shocks to the price of robots. As robots become more important

[4] While this paper includes the correlations in the data, the purpose of this paper is not to match empirical moments, but to explore the effects of robots in a business cycle model.

in the economy and as shocks to the price of robots play a more important role in business cycle fluctuations, the relationship between labor's share and output over the business cycle is likely to weaken further.

[Table 3 about here.]

The impulse response functions shown in Figures 2–5 suggest that the presence of robots magnifies the response of output, employment, and labor's share to shocks. This is also reflected in Table 3, which shows the volatility of output, employment, and labor's share in the model driven by all four stochastic processes. The volatility of all three variables increases when robots are introduced, and the volatility increases further when robots become more substitutable with human labor. The reason is that at the business cycle frequency, the availability of robots affords the firms and the household a greater degree of flexibility in adjusting their allocation decisions. This allows agents to utilize human labor when robots are relatively scarce, and switch to robots when they are relatively abundant or when human labor is relatively more expensive due to nominal rigidity. This in turn leads to more volatile behaviors in output, employment, and labor's share.

This is an intuitive yet powerful result. It is intuitive because substitutable robots allow firms and the household to adjust output by adjusting robot input rather than human labor input. It is powerful because it suggests policy implications—policies aimed to stabilize output may no longer have the same impact at stabilizing employment and vice versa. This point will be explored in greater detail in the following section.

Monetary Policy Experiments

How does the presence of robots affect the monetary authority's decisions regarding monetary policy? This paper does not attempt a complete analysis of optimal monetary policy; instead it will identify some general principles that may inform future research.

Panel (a) of Figure 6 shows output-inflation volatility frontiers for the three scenarios. Each frontier shows the standard deviation of inflation and output for alternative values of ρ_π for that scenario, fixing the value of ρ_y at its baseline value of 0.10. The standard deviations are those implied by the transition equations produced by the first-order perturbation method used to solve the model. The figure shows that the introduction of robots presents the monetary authority with a less favorable trade-off between inflation and output volatility. As discussed in the previous section, more substitutable robots prompts a re-optimization behavior that leads to a greater level of volatility. Consequently a given value for ρ_π produces a larger standard deviation of output and inflation the more substitutable are robots for human labor.

More subtly, the flattening of the output-inflation volatility frontier as the elasticity of substitution between robots and human labor increases implies that a monetary authority concerned with minimizing a loss function including the weighted average of output and inflation volatility would choose a smaller coefficient on inflation in its policy rule. The slope of the volatility frontier is flatter for a given value of ρ_π when the elasticity of substitution between robots and human labor is larger. This means that a further decrease in inflation variability comes at a larger marginal cost in terms of output variability. Suppose the choice of ρ_π reflects the optimizing choice of a monetary authority that minimizes a loss function with a given weighting of inflation and output variability. This monetary authority would respond to the increased marginal cost of inflation reduction introduced by the presence of robots by choosing a smaller value of ρ_π, moving up and to the left along the volatility frontier.

Panel (b) of Figure 6 shows output-inflation volatility frontiers for different values of ρ_y holding ρ_π at its baseline value of 1.49. The panel shows the same deterioration in the output-inflation variability trade-off as in panel (a). While it is not apparent in the figure, there is a slight flattening in these volatility frontiers as in panel (a), implying that an optimizing monetary authority would choose a point on the frontier further up and to the left—implying a larger value of ρ_y—in the strong robot case. In sum, monetary authorities whose objective is to minimize a weighted average of output and inflation volatility would adopt a policy rule with a smaller emphasis on stabilizing inflation and a larger emphasis on stabilizing output in a world where the elasticity of substitution between robots and human labor is larger.

A final implication for monetary policy involves the relationship between the volatility of employment and output. Existing business cycle models do not typically distinguish between a monetary authority's interest in stabilizing output and stabilizing employment. These objectives are assumed to be tightly connected by a "divine coincidence" similar to that identified by Blanchard and Galí (2007) in reference to the inflation and output gap stabilization objectives. In the previous section, however, it is shown that the presence of robots reduces the correlation between output and employment. This has implications for monetary policy.

Figure 7 shows the standard deviation of output and employment implied by the scenarios of the model when the coefficient on inflation in the monetary policy rule is set at its baseline value and the coefficient on output is varied. In all scenarios a larger coefficient on output reduces the standard deviation of employment along with that of output. However, in the two robot scenarios the standard deviation of employment is larger for any given standard deviation of output. Furthermore, if one imagines drawing a horizontal line at a given value of σ_n one sees that the monetary authority would have to adopt a higher value of ρ_y to achieve a given amount of employment stabilization in a world with highly substitutable robots. Thus to the extent that monetary authorities value employment stabilization apart from output stabilization, the presence of robots requires a greater commitment to output stabilization.

[Figure 6 about here.]

[Figure 7 about here.]

Conclusion

This paper explores how the inclusion of human labor-replacing capital, or robots, affects the relationship between output, employment, and labor's share of income in a New Keynesian model. At the business cycle frequency, a shock lowering the price of traditional capital causes labor's share to rise while a shock lowering the price of robot capital causes labor's share to fall. A positive shock to total factor productivity causes labor's share to fall, largely due to interactions between robots and nominal rigidities. All of these effects are greater the higher is the elasticity of substitution between robots and human labor. An expansionary monetary policy shock increases labor's share in the short run, but when robots and human labor are sufficiently substitutable, the labor's share dips below its steady state before returning to its steady state.

The presence of robots weakens the correlation between output and labor's share, primarily by weakening the correlation between output and employment. This effect is larger when robots become more substitutable with human labor and when shocks to the price of robot investment goods take on a greater role in the economy. The presence of robots also increases volatility of output, inflation, and employment.

Finally, the presence of robots has implications for monetary policy. In a world where robot capital plays a more prominent role, this model suggests that monetary policymakers seeking to stabilize output and inflation would need to adopt a monetary policy rule that places less emphasis on inflation and more emphasis on output. A monetary policymaker with a separate interest in stabilizing employment would need to emphasize the stabilization of output to an even greater extent.

This paper is exploratory in nature and could be extended in several ways. First, estimation of model parameters in the style of Smets and Wouters (2005) would produce more precise and model-consistent estimates of the effects of robotization. In addition, while the volatility trade-offs under a Taylor-type monetary policy rule are suggestive, the implications for monetary policy could be examined in more detail within an optimal monetary policy framework. These extensions are left for future research.

Online Supplemental Appendix

A supplemental appendix is available online.

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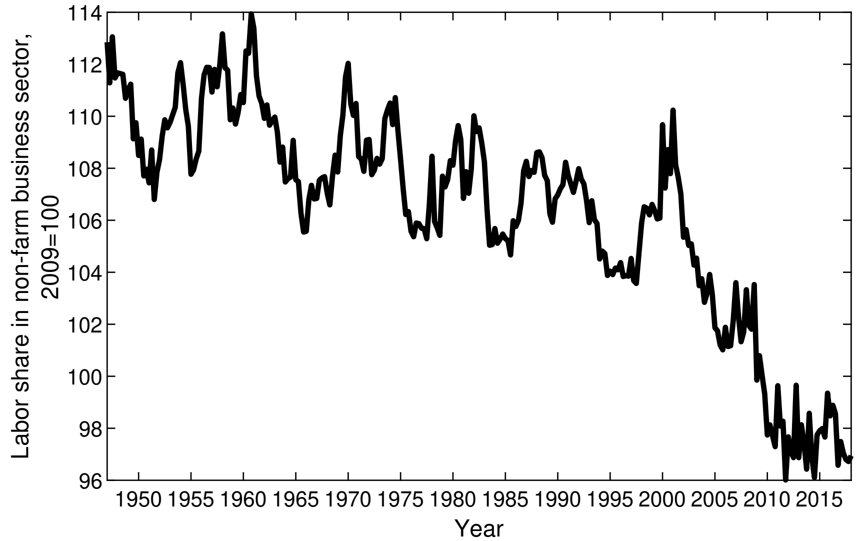

Figure 1: Non-farm business sector labor's share. Source: Bureau of Labor Statistics; series PRS85006173; last assessed on February 1, 2019; https://fred.stlouisfed.org/series/PRS85006173.

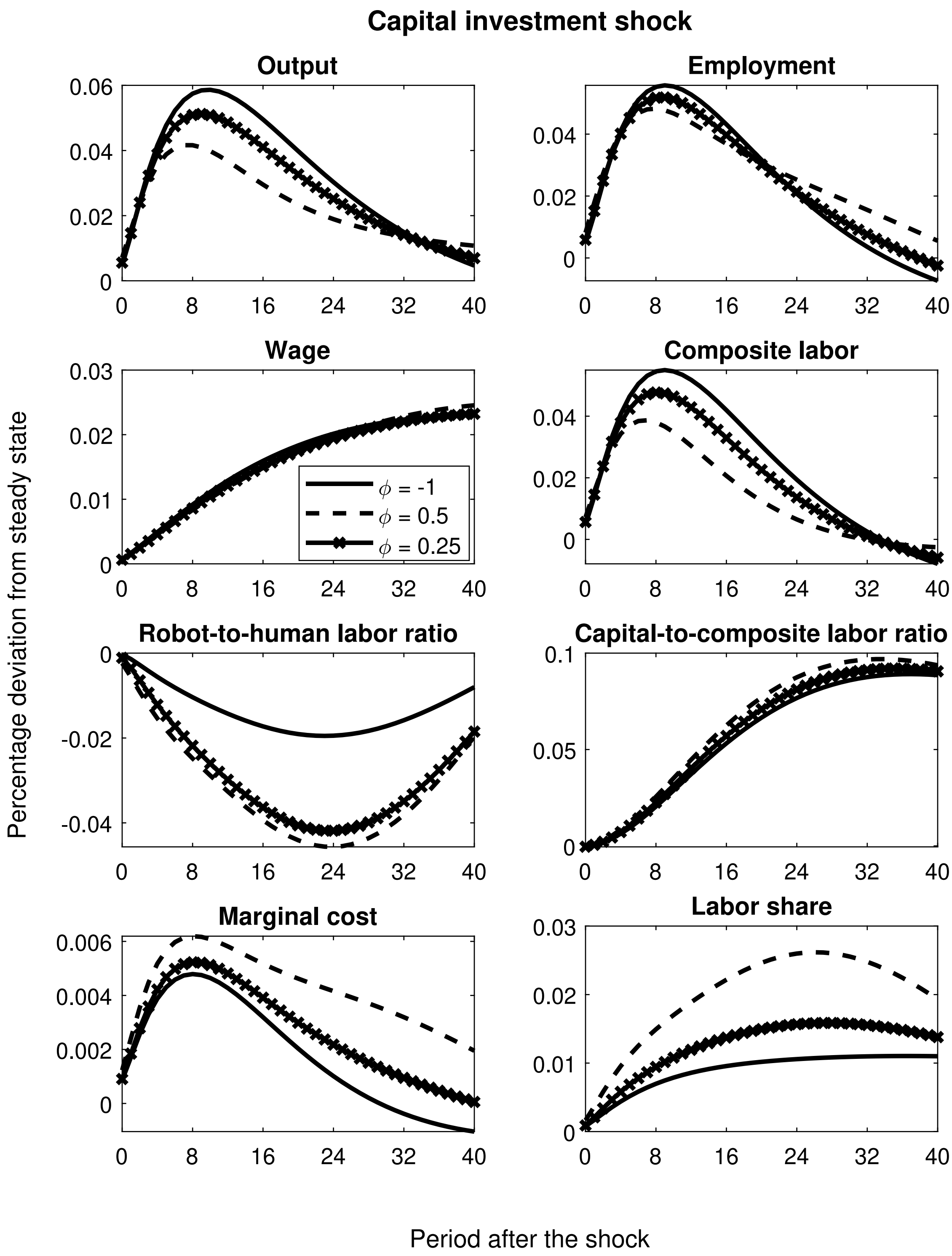

Figure 2: New Keynesian model, capital investment-specific shock. The plain solid line represents the specification in which $\alpha = \phi = -1$; the solid line with $\times$ represents the specification in which $\alpha = -1$ and $\phi = 0.25$; the dashed line represents the specification in which $\alpha = -1$ and $\phi = 0.5$. Source: Own calculations based on the calibrated parameters.

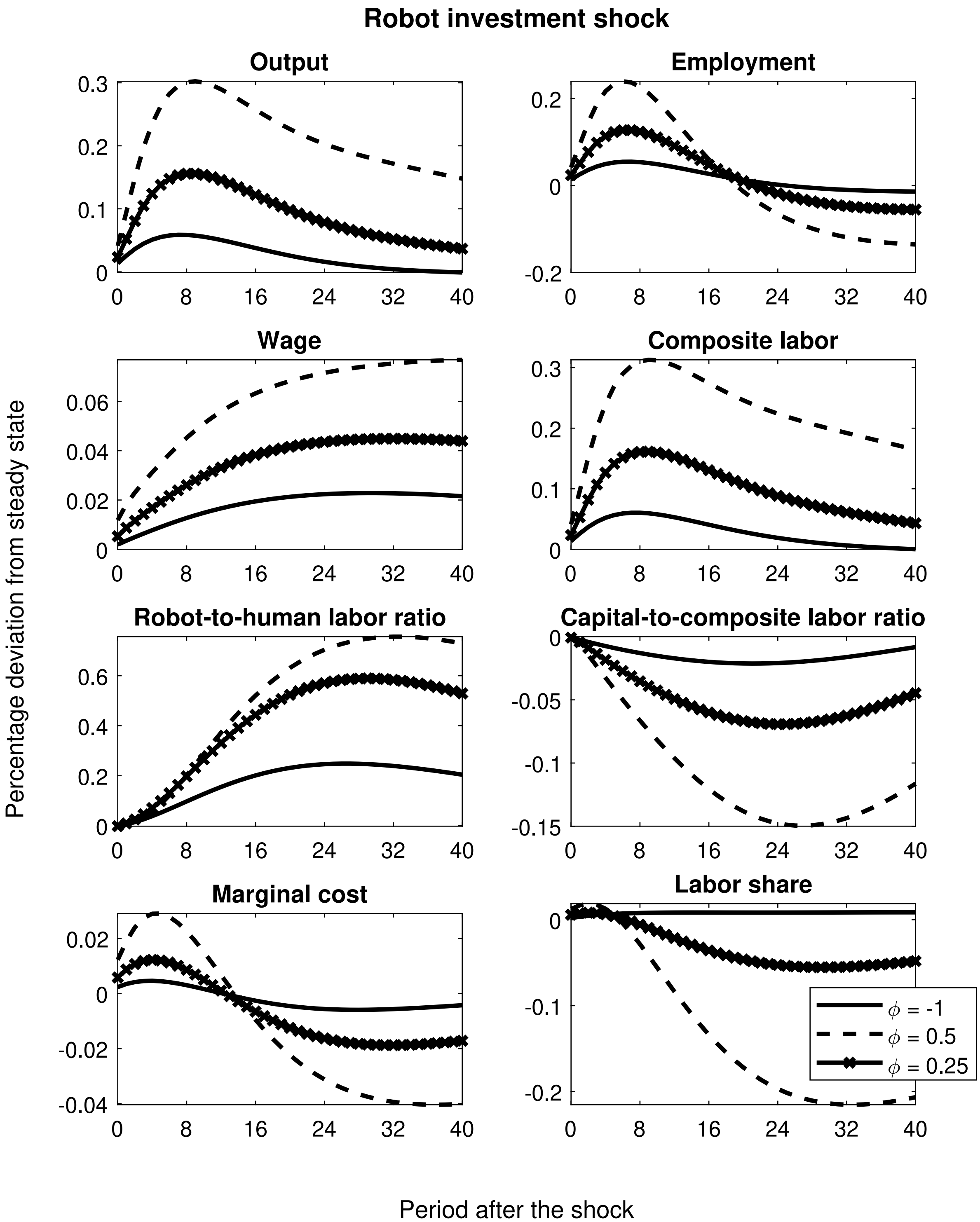

Figure 3: New Keynesian model, robot investment-specific shock. The plain solid line represents the specification in which $\alpha = \phi = -1$; the solid line with $\times$ represents the specification in which $\alpha = -1$ and $\phi = 0.25$; the dashed line represents the specification in which $\alpha = -1$ and $\phi = 0.5$. Source: Own calculations based on the calibrated parameters.

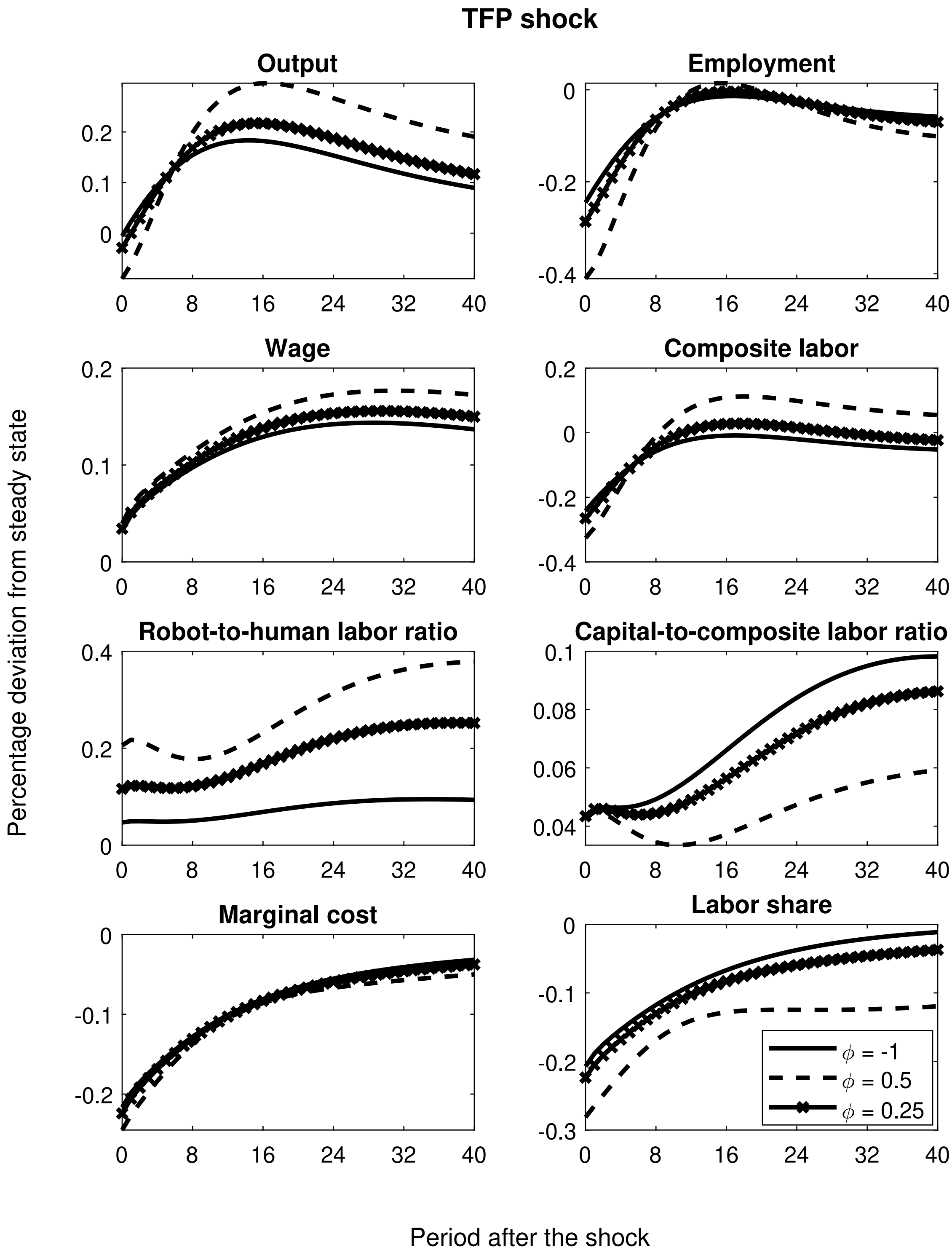

Figure 4: New Keynesian model, TFP shock. The plain solid line represents the specification in which $\alpha = \phi = -1$; the solid line with $\times$ represents the specification in which $\alpha = -1$ and $\phi = 0.25$; the dashed line represents the specification in which $\alpha = -1$ and $\phi = 0.5$. Source: Own calculations based on the calibrated parameters.

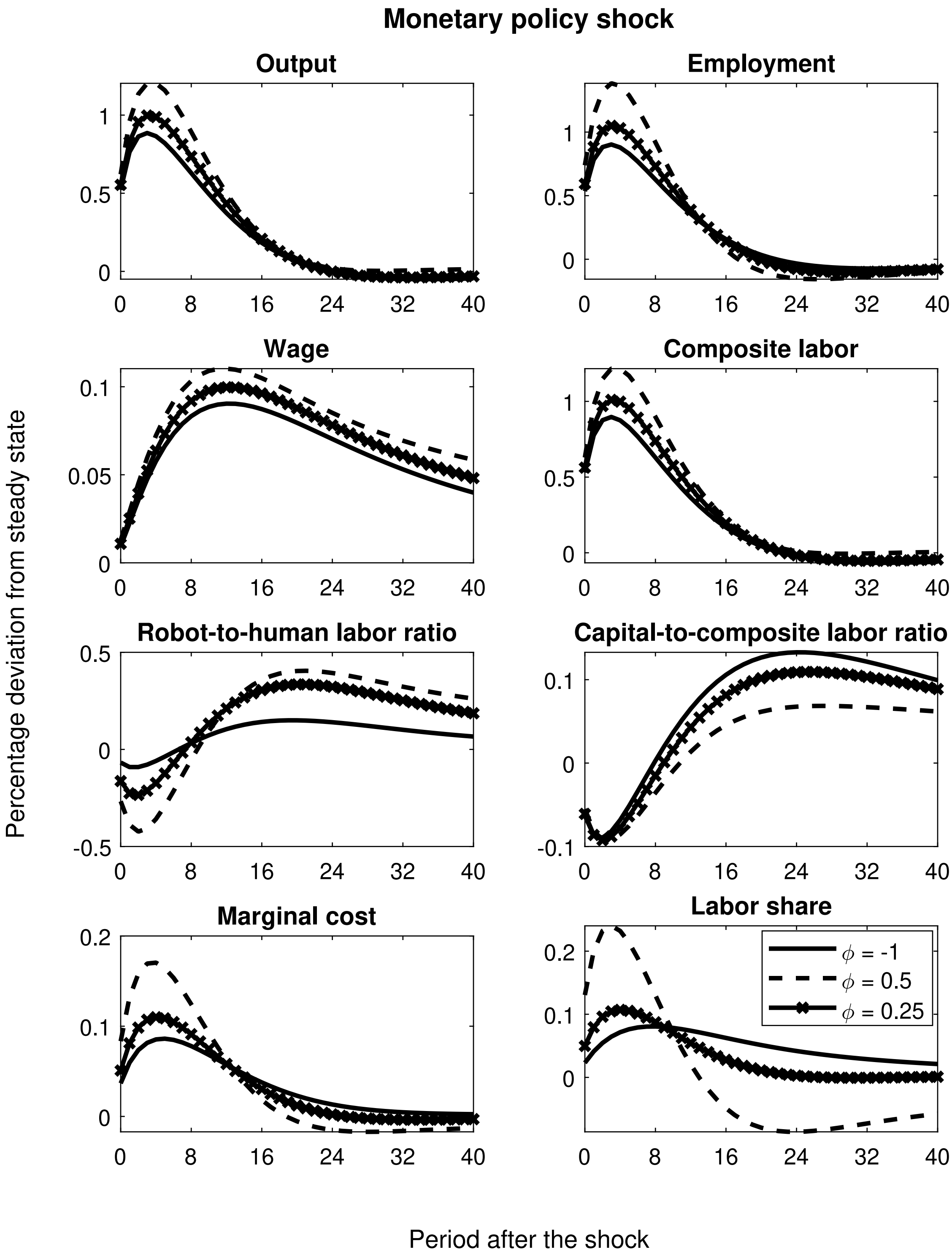

Figure 5: New Keynesian model, nominal interest rate shock. The plain solid line represents the specification in which $\alpha = \phi = -1$; the solid line with $\times$ represents the specification in which $\alpha = -1$ and $\phi = 0.25$; the dashed line represents the specification in which $\alpha = -1$ and $\phi = 0.5$. Source: Own calculations based on the calibrated parameters.

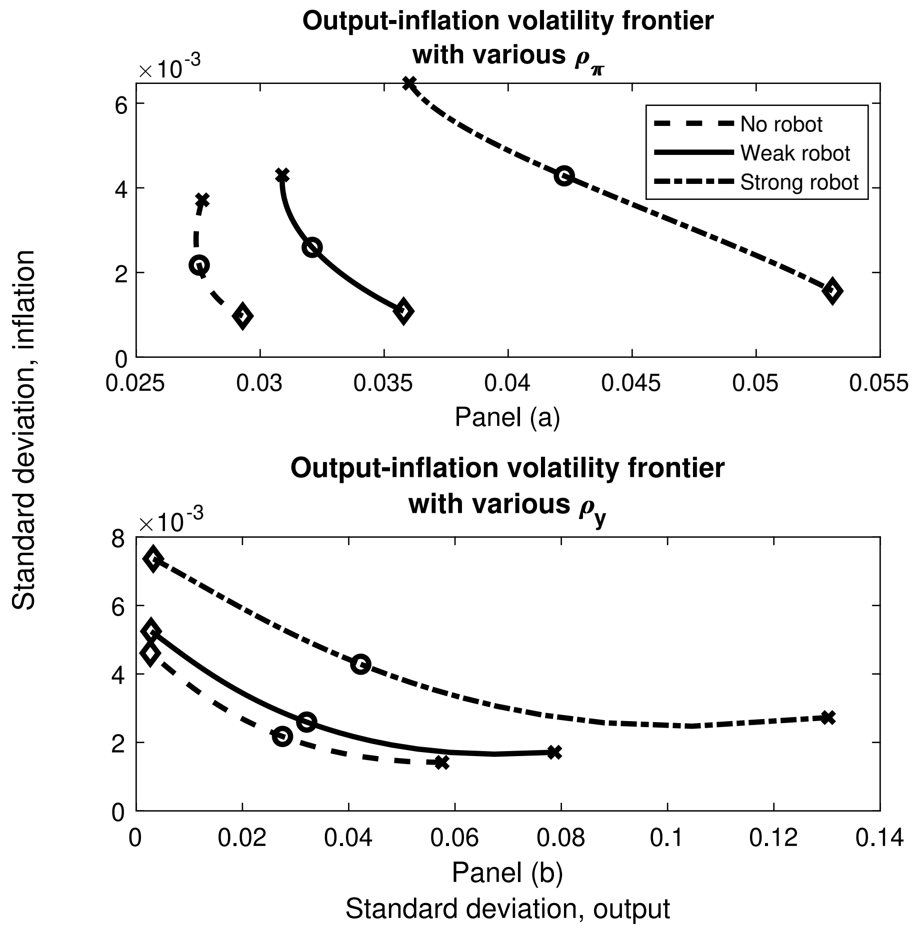

Figure 6: Output and inflation volatility trade-off. In panel (a), the $\times$ markers denote $\rho_\pi = 1$; $\circ$ markers denote $\rho_\pi = 1.49$; the $\diamond$ markers denote $\rho_\pi = 4$. In panel (b), the $\times$ markers denote $\rho_y = 0$; $\circ$ markers denote $\rho_y = 0.1$; the $\diamond$ markers denote $\rho_y = 3$. Source: Own calculations based on the calibrated parameters.

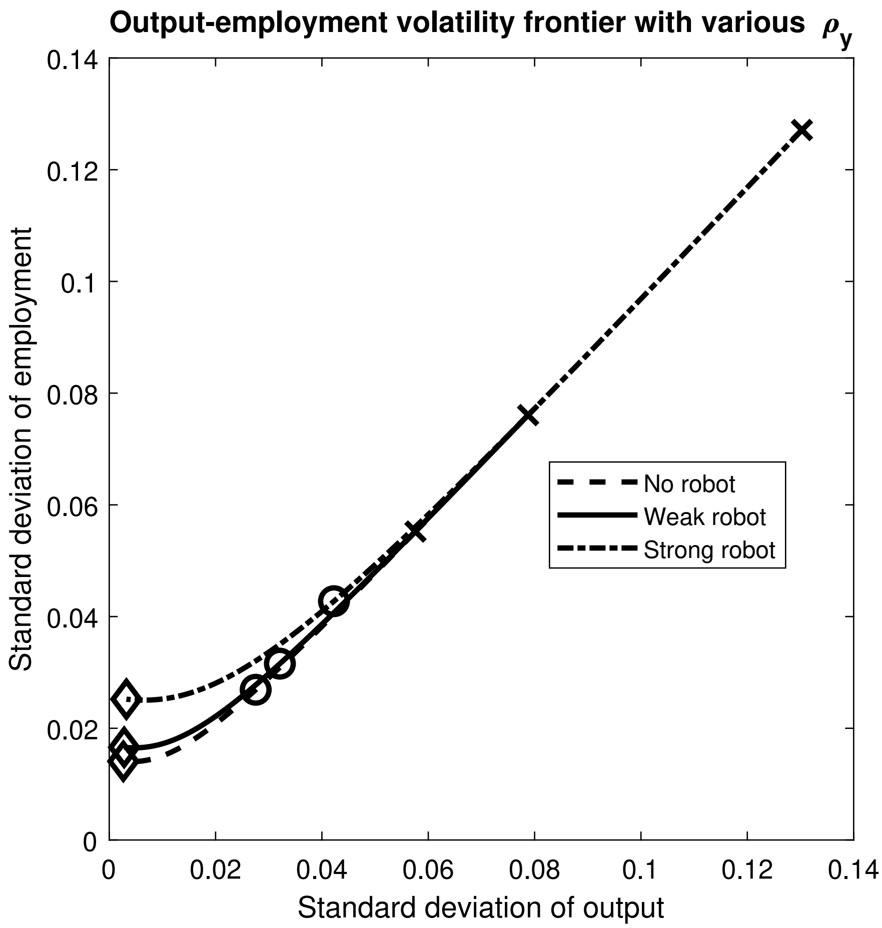

Figure 7: Output and employment volatility trade-off by varying monetary policy's sensitivity to deviations in output ρ_y while holding its sensitivity to deviations in inflation at the baseline value. The $\times$ markers denote $\rho_y = 0$; $\circ$ markers denote $\rho_y = 0.1$; the $\diamond$ markers denote $\rho_y = 3$. Source: Own calculations based on the calibrated parameters.

Table 1: Benchmark calibration

Parameter	Value	Description
β	0.99	Discount rate
γ	1.95	Coefficient of relative risk aversion
σ	2.88	Labor elasticity
κ_n	0.25	Disutility of labor
h	0.44	Habit formation
δ_k	0.020	Depreciation rate, capital
δ_a	0.040	Depreciation rate, automaton
θ_k	0.5459	Capital weight in the CES production
θ_a	0.1251	Robot weight in the CES production
κ_k	5.36	Curvature of capital investment adjustment cost
κ_a	5.36	Curvature of robot investment adjustment cost
ν_k	0.31	Curvature of capital utilization cost
ν_a	0.31	Curvature of robot utilization cost
λ_y	0.91	Frequency of non-price adjustment
η_y	0.34	Price stickiness index
λ_n	0.89	Frequency of non-wage adjustment
η_n	0.75	Wage stickiness index
ε_y	1.23	Steady state markup, intermediate good
ε_n	1.15	Steady state markup, labor union
π^*	1.00	Inflation target
ρ^a	0.9652	Robot investment shock persistence
ρ^k	0.9662	Capital investment shock persistence
ρ^z	0.9854	Productivity shock persistence
ρ^{mp}	0.7063	Monetary policy shock persistence
ρ^R	0.90	Taylor rule interest rate inertia
ρ^π	1.49	Taylor rule inflation weight
ρ^y	0.1	Taylor rule output gap weight
ς^a	0.0079	Standard deviation of robot investment innovations
ς^k	0.0035	Standard deviation of capital investment innovations
ς^{mp}	0.0007	Standard deviation of monetary policy innovations
ς^z	0.0023	Standard deviation of productivity innovations

Source: Own calculations using data from U.S. Bureau of Economic Analysis (2019); U.S. Bureau of Labor Statistics (2019a,b) and adopted from Smets and Wouters (2005) and Justiniano et al. (2010).

Table 2: Data and model correlation coefficients

Correlation coefficients	TFP	Capital	Robots	Monetary	All shocks	
Labor's share and output					−0.220	Data
	−0.623	0.322	0.399	0.722	0.169	$\phi = -1$
	−0.730	0.666	−0.598	0.992	0.085	$\phi = 0.25$
	−0.850	0.513	−0.787	0.706	−0.220	$\phi = 0.5$
Output and employment					0.873	Data
	−0.605	0.929	0.902	0.998	0.844	$\phi = -1$
	−0.567	0.869	0.410	0.990	0.806	$\phi = 0.25$
	−0.567	0.449	−0.143	0.960	0.613	$\phi = 0.5$

Notes: Column 1 indicates the correlation coefficient being examined. Columns 2 to 5 represent when the model is driven by individual shocks; column 6 shows the correlation coefficients when model is driven by all shocks. There are four rows for each correlation coefficient. Row 1 indicates the correlation in the data; rows 2 to 4 show the correlation for each of the no robot $\phi = -1$, weak robot $\phi = 0.25$, and strong robot $\phi = 0.5$ specifications. Source: Own calculations based on calibrated parameters and data from U.S. Bureau of Economic Analysis (2019); U.S. Bureau of Labor Statistics (2019a,b).

Table 3: Data and model volatility

	$\phi = -1$	$\phi = 0.25$	$\phi = 0.5$	Data
Output	0.028	0.032	0.042	0.0162
	(1.000)	(1.000)	(1.000)	(1.000)
Employment	0.027	0.032	0.043	0.0189
	(0.976)	(0.983)	(1.012)	(1.167)
Labor's share	0.007	0.008	0.021	0.0107
	(0.245)	(0.256)	(0.505)	(0.661)

Notes: Column 1 indicates the standard deviation being examined. Columns 2 to 4 corresponds to the no robot $\phi = -1$, weak robot $\phi = 0.25$, and strong robot $\phi = 0.5$ specifications respectively. Column 5 shows the data volatility. The numbers in parentheses are the relative (with output) standard deviations. Source: Own calculations based on calibrated parameters and data from U.S. Bureau of Economic Analysis (2019); U.S. Bureau of Labor Statistics (2019a,b).