The Role of Uncertainty in the Joint Output and Employment Dynamics

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Abstract

This paper examines the role uncertainty plays in the joint dynamics between output and employment. To account for the periodic negative co-movement between output and employment observed in the data, I develop a dynamic stochastic general equilibrium model with search and matching frictions in the labor market and an intensive labor margin. The model is driven by productivity and time-varying volatility shocks. The uncertainty agents face is captured by time-varying volatility. Labor market search frictions generate costly labor adjustment. When an uncertainty shock hits the economy, firms reduce the number of vacancies because they are reluctant to make costly adjustments along the extensive margin. Instead, firms require more effort from their employees. An economy hit by an uncertainty shock and a positive productivity shock simultaneously can thus experience a negative co-movement between output and employment as periodically observed in the data.

Keywords: Search and matching; uncertainty; employment dynamics; intensive labor margin.
JEL Classification: E24; E32.

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1 Introduction

One of the most salient macroeconomic relationships in the post-war era has been the co-movement of output and employment. Indeed, the inverse relationship between output and unemployment was first quantified in Okun (1962), and this robust empirical regularity has come to be known as Okun's law. Moreover, conventional macroeconomic models, including search-and-matching models, predict strong positive co-movement between output and employment. All of these make the periodic negative co-movements puzzling.[1]

In this paper, I explore the role of aggregate uncertainty plays in the joint dynamics between output and employment. To do so, I develop a dynamic stochastic general equilibrium model that features search-and-matching frictions in the labor market as pioneered by Merz (1995) and Andolfatto (1996) and incorporate an intensive labor margin.[2] The main driving force of the model is a productivity process. I capture the notion of aggregate uncertainty that agents face with time-varying volatility to the productivity process. When an uncertainty shock arrives, firms become more unsure of their future prospects. Labor market search friction introduces costly labor adjustment; given that firms want to avoid costly mistakes, they turn to the intensive margin by requiring more effort from their employees and reduce the number of vacancies posted. The intensive margin of employment partially makes up for the decrease in output while employment falls. Therefore, if the economy experiences an uncertainty shock and a positive productivity shock simultaneously, output may increase but employment may fall. This allows for the periodic negative co-movement observed in the data.

This paper is related to the contemporaneously developed work by Leduc and Liu (2016), and subsequently by Fasani and Rossi (2018). However, this paper abstracts from nominal rigidities and focuses on the joint dynamics between output and employment.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 discusses the calibration strategy and details the computational strategy. Section 4 presents the results. Section 5 concludes.

2 Model

This section describes the model. To simplify notation, I assume symmetry among households and firms. The detailed description of the model is in A.

2.1 Labor Market

The labor market is subject to search friction. ρ_n fraction of employed workers are exogenously separated. Let n_{t-1} be the number of employed workers in period $t-1$. The number of job seekers is:

$$u_t = 1 - (1-\rho_n)n_{t-1}.$$

Let v_t be the number of vacancies and m_t matches formed. I assume a Cobb-Douglas matching function:

$$m_t = m_n u_t^{\mu} v_t^{1-\mu},$$

where m_n is the efficiency parameter and $\mu \in (0,1)$ is the match elasticity with respect to job seekers.

The law of motion for aggregate employment is:

$$n_t = (1-\rho_n)n_{t-1} + m_t.$$

Given the quarterly timing, I allow a worker who is exogenously separated at the beginning of a period to—(1) join the pool of job seekers; (2) form a match with an employer; and (3) produce output—within the same quarter.

The job finding rate and the vacancy fill rate are $s_t = \frac{m_t}{u_t}$, and $q_t = \frac{m_t}{v_t}$, respectively.

[1]For example, the "jobless recoveries" after the 1990 and the 2001 recessions.

[2]See, for example, Trapeznikova (2017) and Galí and van Rens (2010).

2.2 Households

The economy consists of a continuum of households. I follow Merz (1995) and Andolfatto (1996) in assuming that each household consists of an infinite number of members. There is perfect risk-sharing among household members. Employed workers receive income $w_t h_t$, the per-effort wage rate times the effort exerted. Unemployed workers receive unemployment insurance b from the government.

Let c_t denote consumption and h_t the intensive labor margin. The households' objective function is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{c_{t+s}^{1-\gamma} - 1}{1-\gamma} - \kappa_h \frac{h_{t+s}^{1+\phi}}{1+\phi} n_{t+s} \right), \tag{1}$$

where β is the discount factor; γ is the coefficient of relative risk aversion; κ_h is the disutility of effort; $\frac{1}{\phi}$ is the intertemporal elasticity of substitution of effort.

Households own the stock of capital k_t and invest i_t. The capital law of motion is:

$$k_{t+1+s} = (1-\delta) k_{t+s} + \left[1 - \mathcal{S}\left(\frac{i_{t+s}}{i_{t-1+s}} \right) \right] i_{t+s}, \tag{2}$$

where δ is the depreciation rate and $\mathcal{S}(\cdot)$ captures investment adjustment cost. A household chooses capital utilization level μ_t, pays utilization cost $\Psi(\mu_t)$, and receives rental income r_t.

Households maximize their objective function (1) subject to the capital law of motion (2) and the budget constraint:

$$c_{t+s} + i_{t+s} + \Psi(\mu_{t+s}) \leq w_{t+s} h_{t+s} n_{t+s} + (1 - n_{t+s}) b + r_{t+s} \mu_{t+s} k_{t+s} + \Pi_{t+s} - T_{t+s}, \tag{3}$$

where Π_{t+s} is the dividend payments from firms and T_{t+s} is the lump-sum tax to finance the unemployment insurance.

2.3 Firms

Let $\tilde{k}_t$ the units of utilization-adjusted capital used in production. I assume that a firm chooses the same effort for all of its workers. Output is:

$$y_t = a_t \tilde{k}_t^{\alpha} (h_t n_t)^{1-\alpha}, \tag{4}$$

where $\alpha \in (0, 1)$.

The productivity process is:

$$\log a_t = \rho_a \log a_{t-1} + \sigma_{a,t-1} \varepsilon_{a,t}, \tag{5}$$

where ρ_a is the persistence parameter and the innovations $\varepsilon_{a,t}$ are *i.i.d.* $\mathcal{N}(0, 1)$.

The standard deviation of the innovations above, $\sigma_{a,t}$, follows an autoregressive process:

$$\log \sigma_{a,t} = \rho_\sigma \log \sigma_{a,t-1} + (1 - \rho_\sigma) \log \bar{\sigma} + \eta^\sigma \varepsilon_{\sigma,t}, \tag{6}$$

where ρ_σ is the persistence parameter, $\bar{\sigma}$ is the non-stochastic mean of σ_t, η^σ is the standard deviation of the innovations, and $\varepsilon_{\sigma,t}$ is *i.i.d.* $\mathcal{N}(0, 1)$.[3]

Firms acquire utilization-adjusted capital goods $\tilde{k}_t$ from a competitive market at rental rate r_t and post vacancies v_t to attract new workers. The firm's employment law of motion is:

$$n_t = (1 - \rho_n) n_{t-1} + q_t v_t,$$

where q_t is taken as given. The cost of employment adjustment is $\frac{\kappa_v}{2} \left(\frac{q_t v_t}{n_t} \right)^2 n_t$.[4]

Firms discount the future using households' stochastic discount factor. Firms and workers jointly determine wage and effort as described below. Firm chooses $\tilde{k}_t$ and v_t to maximize its lifetime profit:

$$V_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[y_{t+s} - w_{t+s} h_{t+s} n_{t+s} - r_{t+s} \tilde{k}_{t+s} - \frac{\kappa_v}{2} \left(\frac{q_t v_{t+s}}{n_{t+s}} \right)^2 n_{t+s} \right], \tag{7}$$

subject to its employment law of motion.

[3] See Bachmann and Bayer (2013) for a similar treatment of time-varying volatility.
[4] Among others, Rastouil (2018) also utilizes convex adjustment costs.

2.4 Effort and Wage Setting

2.4.1 Effort

Effort is set such that the marginal product equals the marginal disutility of the household; that is:

$$(1-\alpha)\frac{y_t}{h_t} = \frac{\kappa_h h_t^{\phi}}{\lambda_t} n_t. \tag{8}$$

2.4.2 Wages

I assume the firm bargains with its existing workforce collectively, and that all workers with the same productivity receives the same wage.[5]

Let J_t and M_t denote the surplus of a worker to a firm and to a household, respectively. Let w_t^N be the Nash bargaining wage defined as the wage that satisfies the condition $(1-\eta)J_t = \eta M_t$ where η captures the workers' bargaining power. Hall (2005) has argued that w_t^N is too volatile relative to the data, which results in a muted response of employment to productivity shocks. Hall (2005) further points out that any wage between the firms' and workers' reservation wages should be considered a solution to the wage bargaining process. In order to allow the model to generate a realistic employment response to productivity shocks, I adopt the wage rule:

$$w_t = \tau w_{t-1} + (1-\tau) w_t^N, \tag{9}$$

where $\tau \in [0, 1]$ captures wage rigidity.[6]

2.5 Government and Resource Constraint

Government levies a lump-sum tax T_t from the households to finance unemployment insurance $(1 - n_t)b$. Let x^* denote the non-stochastic steady-state value of variable x, I assume the unemployment insurance b satisfies the condition:

$$b + \frac{\kappa_h \frac{{h^*}^{1+\phi}}{1+\phi}}{\lambda^*} = \bar{b}(1-\alpha)\frac{y^*}{n^*};$$

that is, the unemployment insurance is such that the opportunity cost of employment equals a constant fraction of the marginal product of labor in the steady-state.

The resource constraint closes the model:

$$y_t = c_t + \frac{\kappa_v}{2}\left(\frac{q_t v_t}{n_t}\right)^2 n_t + \Psi(\mu_t) + i_t.$$

3 Calibration and Solution

3.1 Calibration

$\alpha = 0.33$; $\beta = 0.99$; δ is 0.026 are standard in the literature. I choose an intertemporal elasticity of substitution of 0.5, implying $\phi = 2$. In the absence of a non-convex adjustment cost, γ plays a key role in agents' response to uncertainty shocks; I set the coefficient of relative risk aversion, γ, to 3.[7] The intensive margin is normalized to 1 in the non-stochastic steady-state. $\eta_k \equiv S''(\cdot) = 2.85$ and $\nu_k \equiv \frac{\Psi''(1)}{\Psi'(1)} = 5.3$ as in Justiniano et al. (2010).

[5] In a multi-worker firm model such as the one examined here, there exists an intra-firm bargaining framework first highlighted by Stole and Zwiebel (1996b) and Stole and Zwiebel (1996a) where a firm bargains with its workers individually. However, given that Krause and Lubik (2013) have shown that intra-frim bargaining has a small business cycle effect, I choose a wage-setting mechanism that does not include this game theoretical aspect.

[6] See Hall (2005) for a discussion of this particular adaptive wage determination process.

[7] While this is higher than the standard business cycle literature, Barsky et al. (1997) find that, based on survey responses, the lower bound of relative risk aversion is 3.8, suggesting that deviating from the usual log-utility might not be unreasonable.

I set the labor market parameters to have steady-state values close to the U.S. from 1969Q1 to 2016Q4. The steady-state job seekers $u^* = 0.157$ which yields the 6.3% measured unemployment rate. I set $q^* = 0.7$ as in den Haan et al. (2000). $\rho_0 = 0.1$ is consistent with Shimer (2005). Both the elasticity of matches to unemployment and the workers' bargaining power are set to 0.5 as routinely done in the literature. The replacement ratio $\bar{b} = 0.73$ follows Mortensen and Nagypál (2007). The rigid wage parameter $\tau = 0.56$ is chosen so the elasticity of market tightness with respect to productivity matches the data.

Lastly, I calibrate these parameters governing the two stochastic processes as part of the model evaluation procedure described below.

Table 1 summarizes the calibrated parameters.

[Table 1 about here.]

3.2 Model Evaluation Strategy

The model is solved by third-order perturbation methods; third-order solutions allow me to simulate the model with second-order perturbations.

I solve, period-by-period, the productivity and volatility required to match data on detrended U.S. GDP and nonfarm payroll. The procedure yields a productivity and a volatility series that together allow my model to match the data output and employment.

Not surprisingly, the resulting productivity and volatility processes depend on the parameter values of ρ_a, ρ_σ, $\bar{\sigma}$, and η^σ. I carry out a "fixed point" algorithm in which I begin with a set of initial parameters.[8] I then estimate the resulting productivity and volatility series and update my parameters. I repeat the process until the parameters are sufficiently close between iterations.[9]

Given that the exercise is designed to allow my model to match data on output and employment by varying volatility, it is important to closely examine the resulting stochastic processes. I will do so in the next section.

4 Results

4.1 Impulse Response Functions

Figure 1 shows the impulse response function to a one standard deviation shock to uncertainty from the non-stochastic steady-state. The impulse response function corroborates with our intuition that vacancy falls and the extensive margin rises following an uncertainty shock.

[Figure 1 about here.]

4.2 Computational Results

The rest of this section is organized as follows. First I will show that, with time-varying volatility, the model employment can replicate the periodic negative co-movements between output and unemployment; without it, the model cannot. Next, I examine the implied productivity, volatility, along with other key labor market variables, to verify the model uses reasonable productivity and volatility series and its internal mechanics generates reasonable labor market dynamics.

[Figure 2 about here.]

Each panel of Figure 2 shows three series. Series (1) (solid line with $\times$s) is the U.S. real GDP or non-farm payroll, HP de-trended with a smoothing parameter 1,600. Series (solid line) represents the model driven

[8] The initial values I had chosen were $\rho_a = 0.859^{1/4}$, $\bar{\sigma} = 6.71\%$, $\eta^\sigma = \log 1.93$—these values were calibrated in Bloom et al. (2018). I picked $\rho_\sigma = 0.859^{1/4}$ to match the persistence of a.

[9] The shortcoming of this model is that $\bar{\sigma} = 0.05$ is larger than what is typically estimated as the standard deviation of the total productivity process. This is likely due to the naive assumption that only two processes drive the economy. Nonetheless, this assumption allows us to focus on the role of uncertainty.

by both productivity and uncertainty processes. Series (3) (dotted line) it shows the counterfactual output and employment without time-varying volatility.

We can make two observations. One, time-varying volatility allows the model to replicate the dynamics of actual output and employment. This is shown by series (1) and (2) overlapping each other in Figure 2. Two, in the absence of uncertainty, employment positively co-moves with output and thus fails to replicate the data.

[Figure 3 about here.]

[Figure 4 about here.]

To verify the model uses reasonable productivity and volatility processes, we turn to Figures 3 and 4. Figure 3 compares the model productivity with TFP computed by Fernald (2014). The correlation coefficient between the two is 0.91. Figure 4 compares the model volatility with three common proxies for uncertainty—corporate profit forecast dispersion, corporate bond spread, and consumer uncertainty.[10] To facility comparison across different measures of uncertainty, I first demean and standardize each. I then compute the 1-quarter centered moving average in order to remove high-frequency fluctuations. We see that the model volatility rises during recessions as documented by the literature. It is also reasonably correlated with the data proxies as seen in Table 2.[11]

[Table 2 about here.]

Figure 5 shows vacancy and job finding rate, the other two key labor market variables in the model. We can see that model's internal mechanism allows it to generate paths for both variables that closely resemble their empirical counterparts when time-varying volatility is included.

[Figure 5 about here.]

To establish the robustness of the results, I explore several alternative specifications of the model parameters. See appendix A for more details.

5 Conclusion

This paper shows the inclusion of aggregate uncertainty allows an otherwise standard model to generate the periodic negative co-movement between output and employment as observed in the data. The parsimonious nature of this model makes it uniquely suited to be adopted into a wide class of macroeconomic models to help us better understand the labor market.

A Supplementary Material

Supplementary material related to this article can be found online.

[10] See A for the construction of these proxies. See Bloom et al. (2018) for a survey of the literature.

[11] Note that proxies of uncertainty tend not to be highly correlated; the correlation between the forecast dispersion and bond spread, for example, is 0.34.

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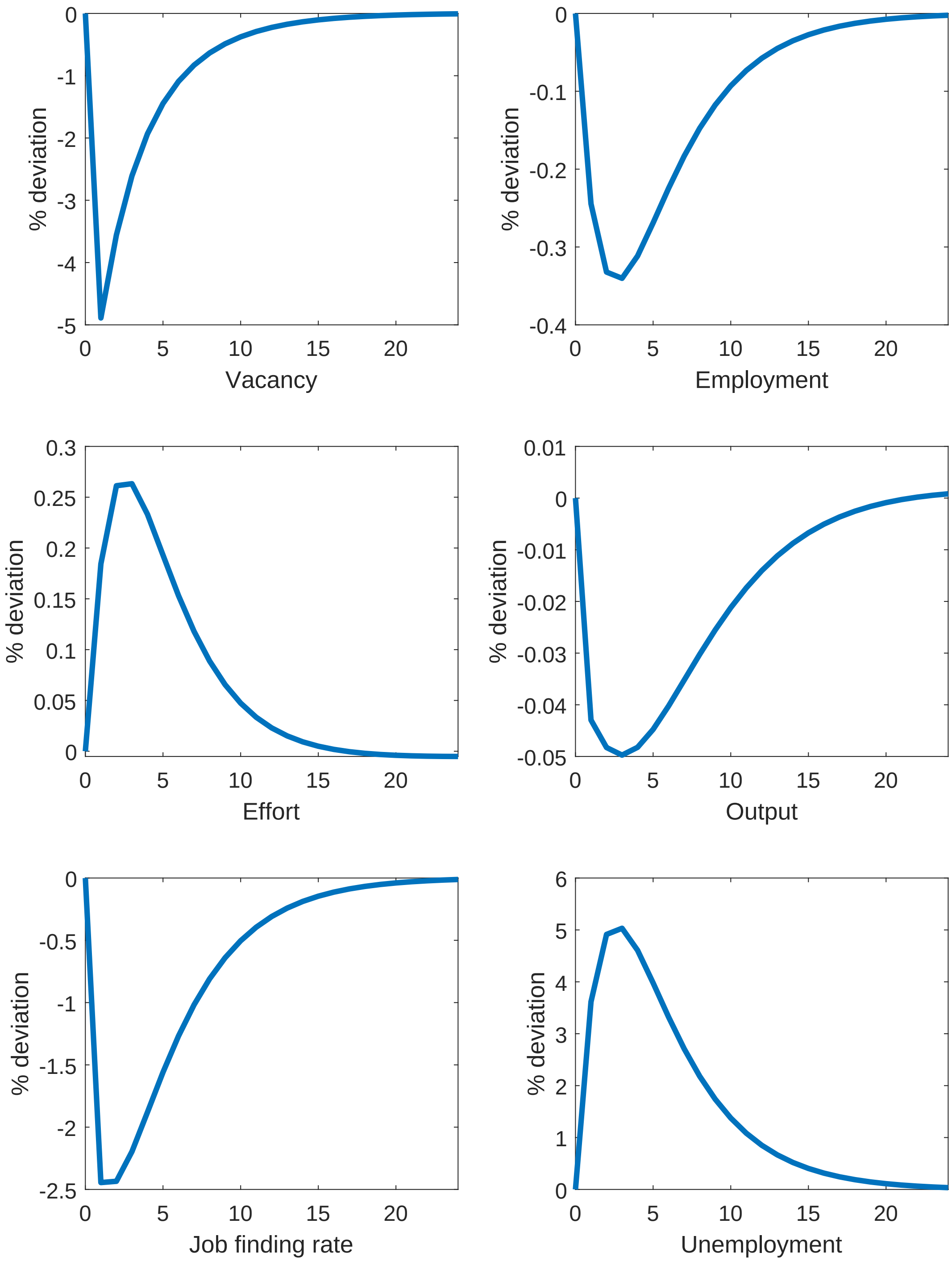

Figure 1: Impulse response of a one standard deviation shock to the time-varying volatility. y-axis is percentage deviation from steady-state. x-axis is quarters.

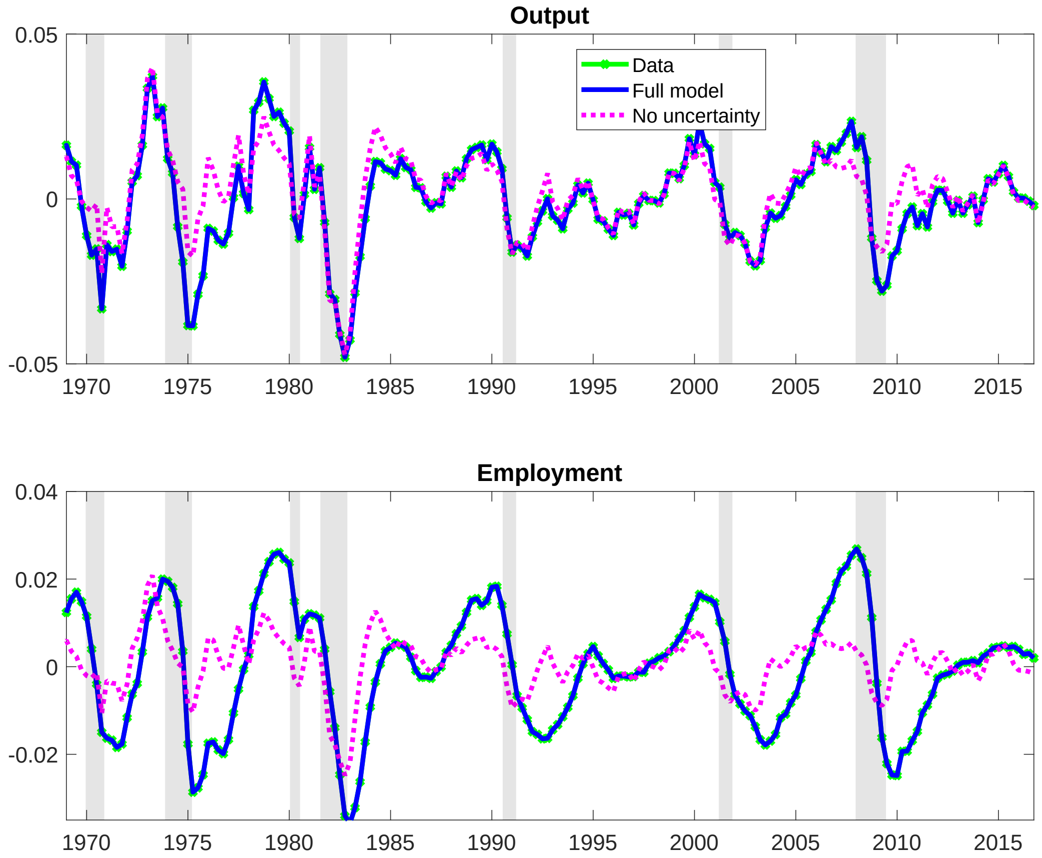

Figure 2: Model and data output and employment.

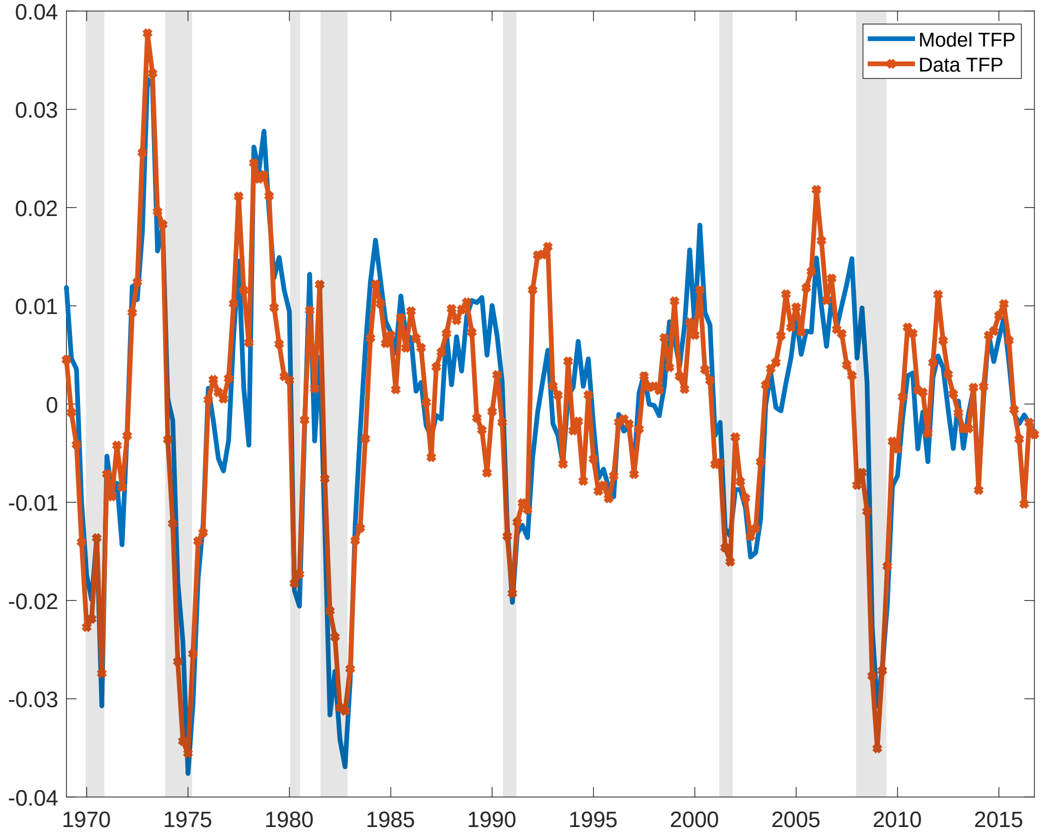

Figure 3: Model-implied volatility and TFP in Fernald (2014).

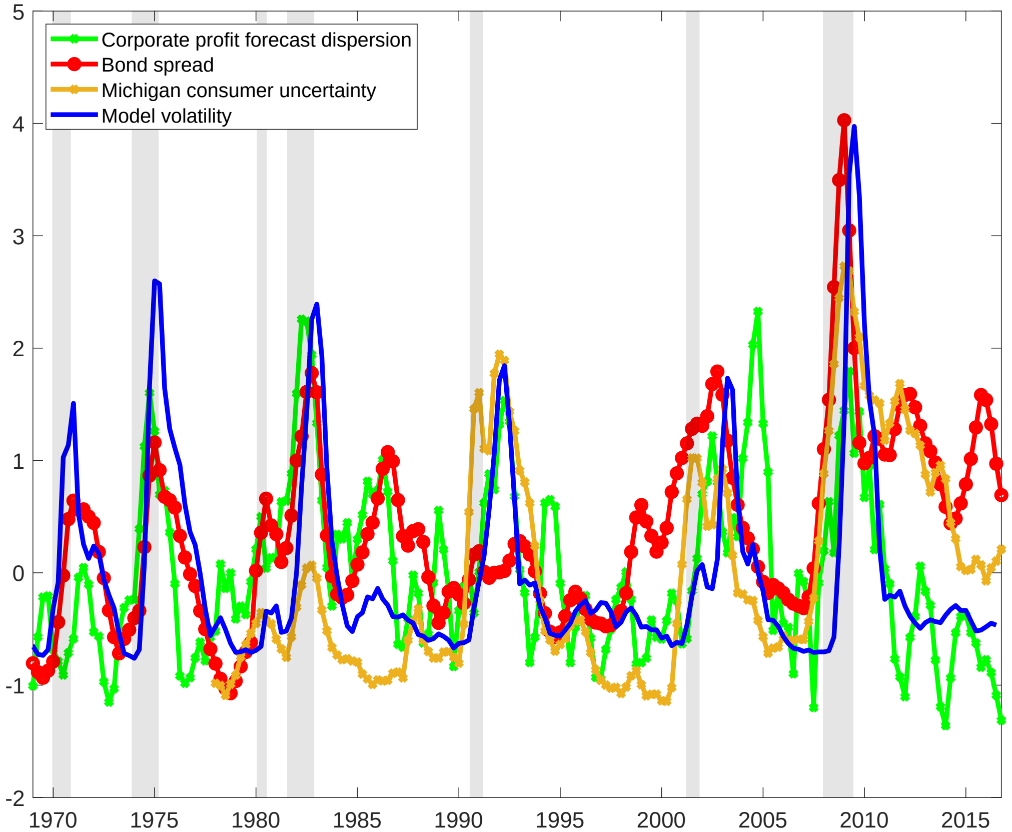

Figure 4: Model-implied volatility and proxies for uncertainty.

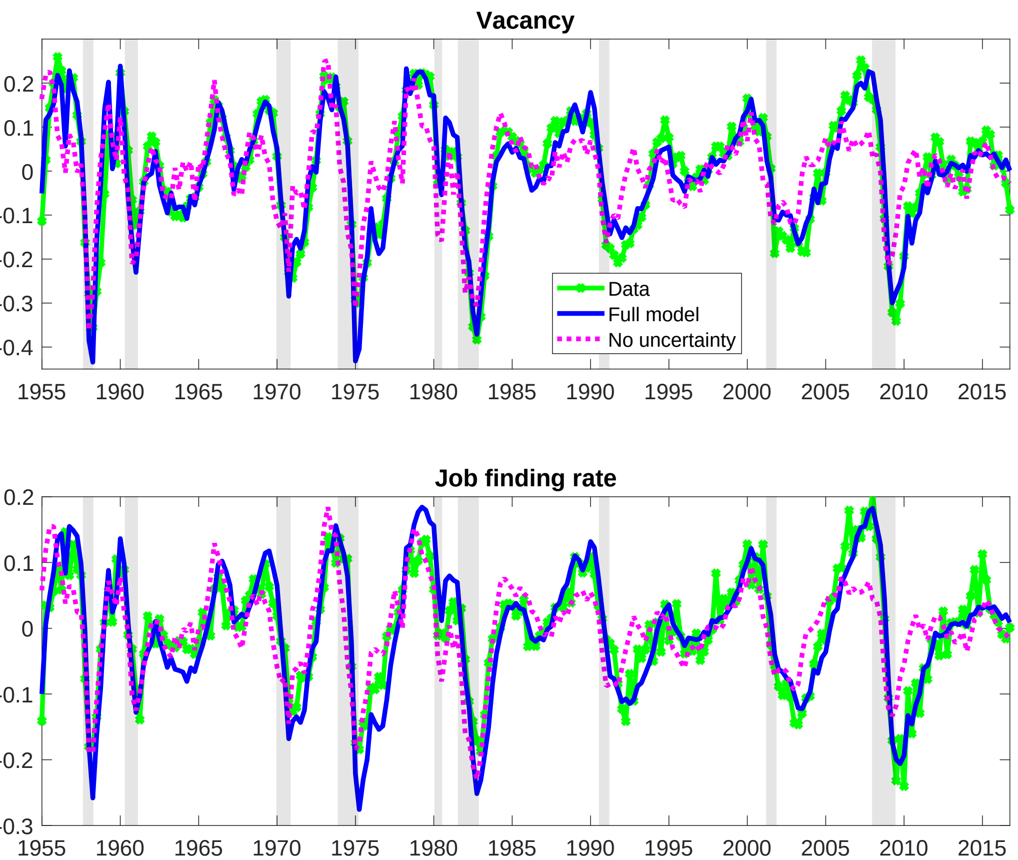

Figure 5: Model-implied vacancy and job finding rate. The data vacancy is downloaded from Regis Barnichon's website computing following Barnichon (2010). The job finding rate is computed by the author using the algorithm specified in Shimer (2012).

Parameter	Value	Description
α	0.33	Capital share
β	0.99	Discount rate
δ	0.026	Depreciation rate
ϕ	2	Elasticity of substitution
γ	3	Risk aversion
$\bar{u}$	0.157	Job seekers
$\bar{q}$	0.7	Vacancy fill rate
ρ_0	0.10	Separation rate
μ	0.5	Elasticity of matches to unemployment
η	0.5	Worker share of match surplus
$\bar{b}$	0.72	Replacement ratio
τ	0.56	Wage index
η_k	2.85	Investment adjustment costs
ν_k	5.3	Elasticity of capital utilization costs
ρ_a	0.80	Productivity process persistence
ρ_σ	0.76	Uncertainty process persistence
$\bar{\sigma}$	0.05	Non-stochastic mean of σ
η^σ	$\log 1.80$	Standard deviation of uncertainty shocks

Table 1: Calibrated parameters. See Section 3.1 for details.

	Bond spread	Forecast dispersion	Consumer uncertainty
Correlation coefficient	0.513	0.441	0.547

Table 2: Correlation with uncertainty proxies